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Velocity Profile Measurements in a Spinning, Cold-Flow Rocket Motor

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I. Introduction

A PROBLEM of considerable experimental and theoretical interest is the description of the interior ballistics of a spinning, solid-propellant rocket motor. This interest has developed since the use of spin stabilized, solid-propellant rocket motors has shown that the actual performance under spinning conditions differed substantially from the performance predicted from nonspinning, solid-propellant rocket motor data.

The purpose of the investigation reported herein was to study the effects of rotation on the flow field inside the chamber of a simulated end-burning rocket motor. In particular, the cold-flow study offered the possibility of the use of a velocity probe for determining the profiles of the axial and tangential velocity components within the chamber of the spinning, cold-flow rocket motor.

II. Apparatus

The apparatus used for this investigation was used in the vortex choking studies conducted by Norton, Farquhar, and Hoffman.[1] Basically, the apparatus consisted of a stationary air-feed chamber and a rotating rocket motor chamber and nozzle. The rocket motor chamber was connected to the air-feed chamber by means of a hollow drive shaft. The rocket motor was supported horizontally and was rotated about its longitudinal axis by an air turbine directly connected to the hollow drive shaft. Air was supplied to the head end of the rocket motor chamber via flow from a set of storage tanks, through a metering orifice and the hollow drive shaft. The air passed through a porous surface to simulate the flow emanating from an end-burning grain. The simulated end-burning grain consisted of a sintered steel plate which was used to produce a large axial pressure drop to minimize the variation of the local mass flow per unit area across the grain surface due to the increasing radial pressure gradient with rotation.

Figure 1 illustrates schematically the general arrangement of the cold-flow rocket motor and the mechanism employed

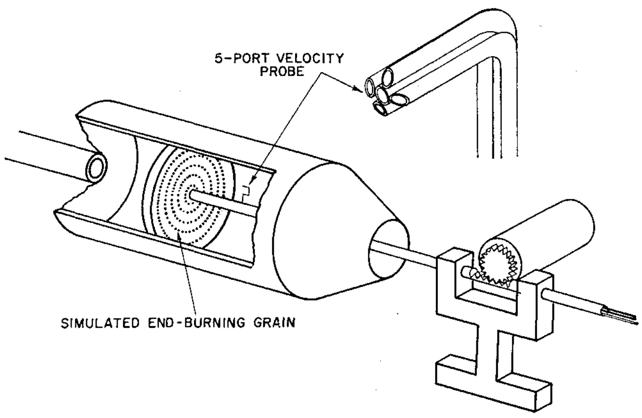

Fig. 1 Cold-flow rocket motor and velocity probe arrangement.

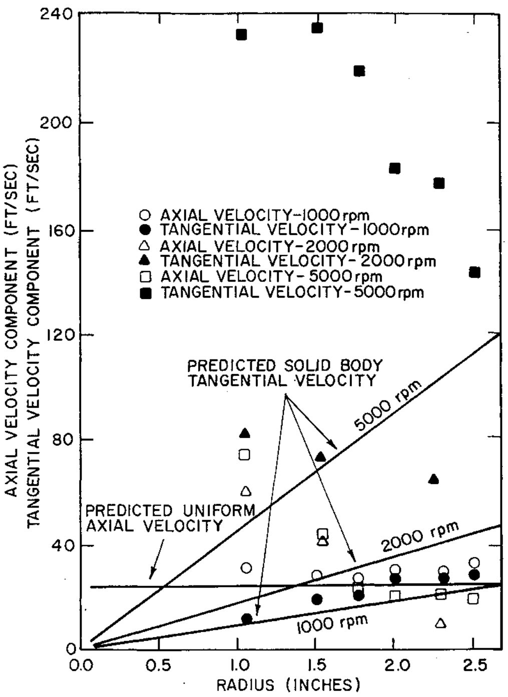

Fig. 2 Velocity profiles — 1.125 in. nozzle.

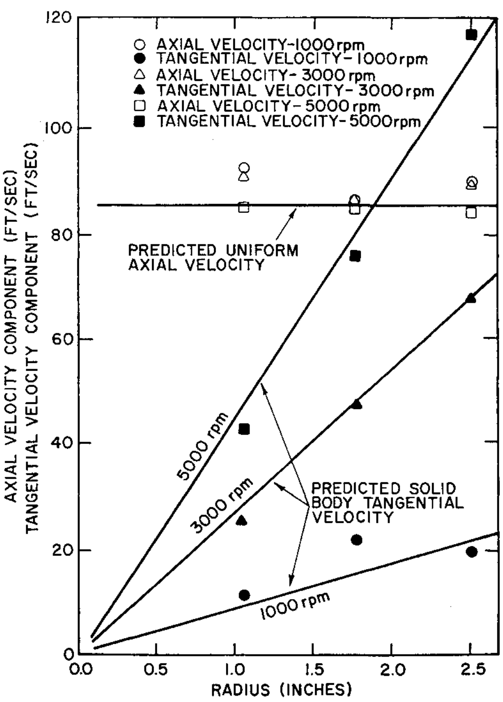

Fig. 3 Velocity profiles — 2.0 in. nozzle.

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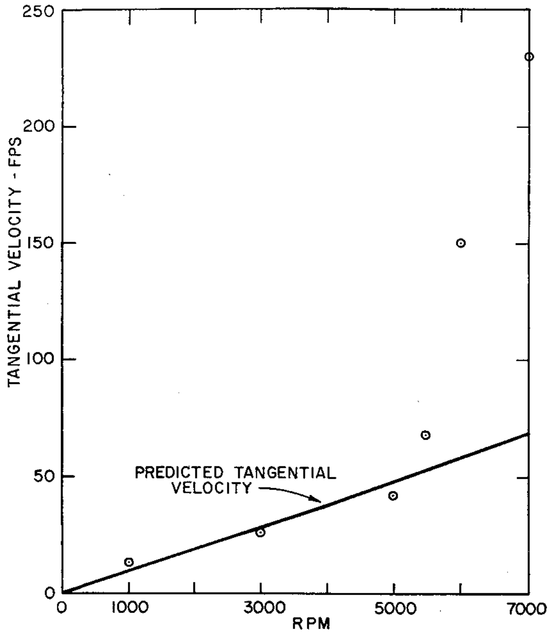

Fig. 4 Tangential velocity at 1.1 in. radius—2.0 in. nozzle.

for positioning a velocity probe within the spinning chamber. A five-port impact tube probe (see inset) was fabricated from 0.020 in. i.d. stainless steel, hypodermic tubing and calibrated for making measurements of the velocity components within a range of ±20° pitch angle and ±50° yaw angle. Measurements of the velocity components as a function of radius were obtained at a distance of 2 in. from the simulated end-burning grain. Data were obtained at selected rotational speeds up to 7000 rpm for 1.125-in. and 2.0-in.-diam nozzle throats with nominal operating chamber pressures of 100 psig and 30 psig. This variation of nozzle contraction ratio from 22.1 to 5.25 provided a nominal axial velocity within the chamber of 25 and 85 fps, respectively.

III. Experimental Results

Figure 2 presents the profiles of the axial velocity component U, and the tangential velocity component V, as functions of radius for the 1.125 in. throat nozzle at rotational speeds of 1000, 2000, and 5000 rpm. The solid lines represent the corresponding profiles for a uniform axial velocity and a solid body tangential velocity predicted for the known values of mass flow and rotational speed. The results indicate that for a rotational speed of 1000 rpm the velocity variations were essentially that of a uniform axial velocity profile and a solid body tangential velocity profile, as expected for the simulated end-burning grain. However, the measured velocity profiles for rotational speeds of 2000 and 5000 rpm did not exhibit these trends, but rather showed a tangential velocity varying inversely with radius. For rotational speeds between 1000 and 2000 rpm, a drastic alteration in the flow-field had occurred.

Figure 3 presents similar results for the measured and predicted axial and tangential velocity profiles obtained for the 2.0 in. throat nozzle at rotational speeds of 1000, 3000 and 5000 rpm. As can be seen, there is good agreement between the measured profiles and those predicted for a uniform axial velocity and solid body tangential velocity.

To explore the influence of higher rotational speeds for the 2.0 in. throat nozzle, additional experiments were conducted to measure the tangential velocity component at a fixed radius of 1.1 in.‡ Figure 4 illustrates the variation of the measured tangential velocity with respect to rotational speed with the solid line representing the variation predicted for a solid body tangential velocity. The data for the 2.0 in. throat nozzle show a large deviation from the predicted solid body tangential velocity for rotational speeds above approximately 5500 rpm. A similar deviation for the data for the 1.125 in. nozzle (Fig. 2) is indicated at rotational speeds above 1300 rpm.

‡ This radius was selected on the basis of the data for the 1.125 in. throat nozzle which show the deviation in the measured and predicted tangential velocity was most pronounced at minimum r.

IV. Conclusions

The significance of these results can be illustrated by a comparison with the recent analytical and experimental study of Dunlap.[2] Dunlap demonstrated the existence of a flow transition from that of a solid body vortex to a free vortex at a certain critical rotational speed. The onset of the flow transition was characterized in terms of a critical Rosby number defined as $(U/2V_m)$, where V_m is the value of the maximum tangential velocity (i.e., at r_{max}) at flow transition. The data presented herein indicate a critical Rosby number for the flow transition of approximately 0.4 for the 1.125 in. nozzle at 1300 rpm and approximately 0.32 for the 2.0 in. nozzle at 5500 rpm which are in reasonable agreement with the value of 0.26 reported by Dunlap.

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Iterative Type Rayleigh-Ritz Method for Natural Vibration Problems

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IN the analysis of eigenvalue problems such as free vibration problems, exact solutions of the governing differential equations are possible only in some special cases. For most problems approximate methods have to be resorted to obtain satisfactorily accurate solutions. Accuracy in approximate methods depends upon several factors such as suitability of admissible function, number of terms used, number of iterations performed, etc. In this Note we present a comparative study of some commonly used approximate methods. This study brings out the relative superiority and effectiveness of the Temple's successive approximation method and indicates that methods based on judicial combination of Rayleigh-Ritz and iterative type methods are of great use in accurate estimation of eigenvalues.

Rayleigh's method is probably the simplest method for the determination of the first natural frequency and is based on equating the kinetic and potential energies associated with an approximating function satisfying geometric boundary conditions. The better the admissible function chosen, the higher the degree of accuracy in the eigenvalue. Temple[1] has de-

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